Diagonal Method for Singly Even Magic Squares

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Introduction

Singly even magic squares are of the form $4\theta + 2$, where θ is a natural number.

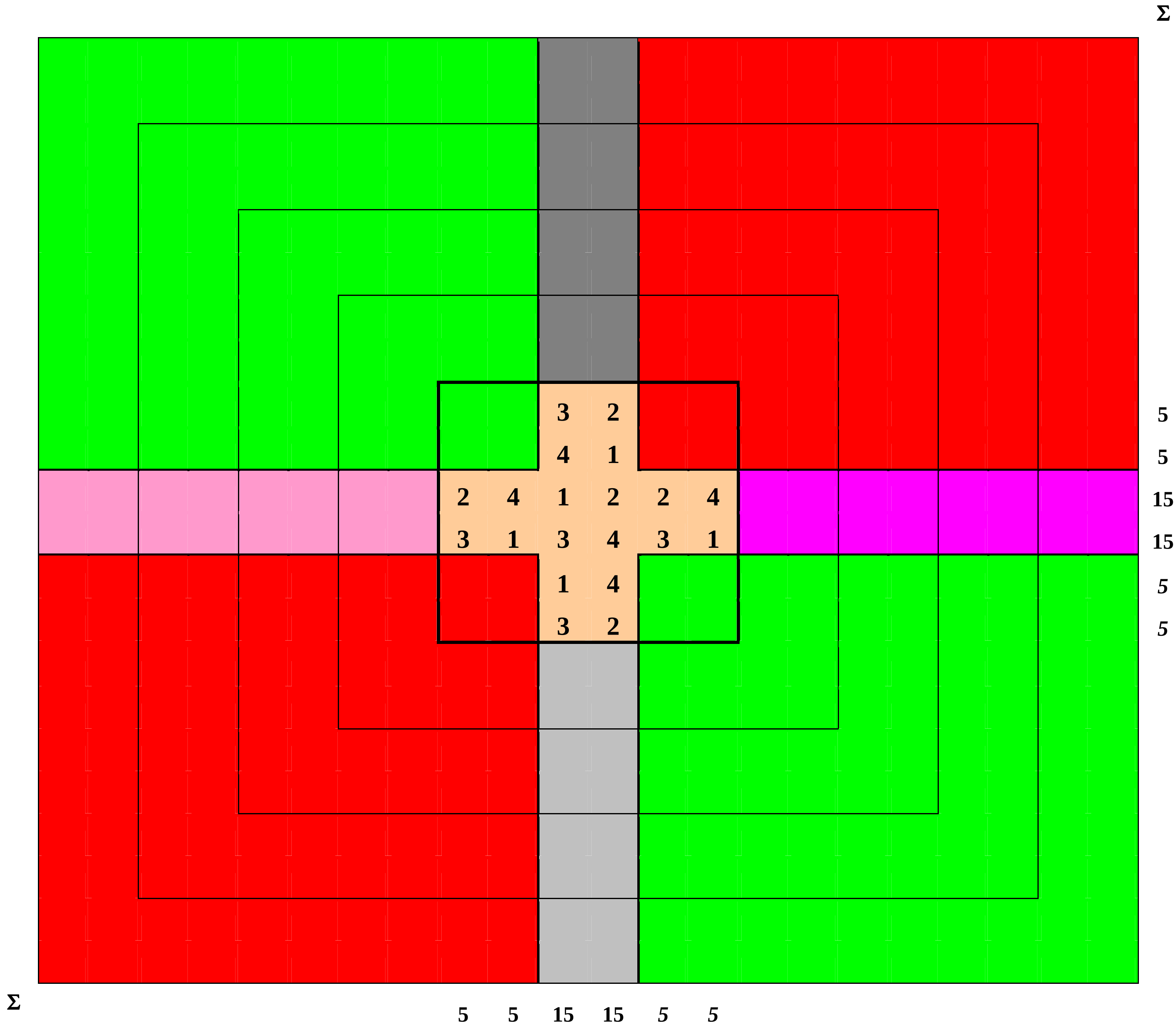
This method follows along the lines of the LUX method. My intent was to develop a pattern that uses the Cartesian coordinate system as a template and has symmetry about the origin. Please bear with me as I develop this theme.

The LUX method requires all squares to be subdivided into 2x2 squares. All rows in the 2x2 squares are to sum to twice the average of the four cell values in the 2x2 square, **σ**. Each 2x2 square would be filled with four sequential values, for simplicity let us label them as 1, 2, 3, 4. Thus **σ** = 5.

As a variation, let us look at what happens when we try to have the diagonals sum to **σ** in the 2x2 square.

Revised: 01/06/2005

Figure 1



If we choose to look at Figure 1 as the Cartesian plane with the origin as the center 2x2 square, the origin and all four quadrants will have the diagonals of their 2x2 squares summing to **σ**. The only sections remaining which do not fulfill this requirement will be the positive and negative x and y axis.

Whatever pattern chosen for Quad II (upper left green area) will also be used in Quad IV (lower right green area), and Quad I (upper right red area) and Quad III (lower left red area) will be filled in with 2x2 squares in reverse order of those in Quad II and Quad IV.

All Y-axis 2x2 squares will have their rows sum to **σ,** except origin. The positive Y-axis beyond the first positive 2x2 square (upper center dark gray area) will reverse the pattern for its counterpart in the negative Y-axis, below the first negative 2x2 square (lower center light gray area).

All X-axis 2x2 squares will have their columns sum to **σ,** except origin. The positive X-axis beyond the first positive 2x2 square (right center pink area) will reverse the pattern for its counterpart in the negative X-axis, prior to the first negative 2x2 square (left center rose area). To save time and add to symmetry one can rotate the gray areas of the Y-axis 90^0 to cover the pink and rose areas but a separate pattern can be used, if desired.

In Figure 1, the innermost square is when θ=1 which produces a 6x6 square. The next larger square is when θ=2 which produces a 10x10 square, et cetera. The 10x10 square, like in the LUX method, is the first square to show the complete pattern for $4\theta+2$ magic squares. From this point, the pattern shown may be extended in all directions. The magic squares will always be subdivided into 2x2 squares so that they can be filled like odd magic squares.

Thus far everything has been very arbitrary. Now only a little less so for the center area shown in Figure 2.

Figure 2

							Σ
			3	**2**			**5**
			4	**1**			**5**
	2	**4**	**1**	**2**	**2**	**4**	**15**
	3	**1**	**3**	**4**	**3**	**1**	**15**
			1	**4**			**5**
			3	**2**			**5**
Σ	**5**	**5**	**15**	**15**	**5**	**5**	

Note that σ = 5

There are only two possible combinations to form patterns which sum to 15 in the figure. They involve:

7 = 3 + 4, **4** = 1 + 3, **4** = 1 + 3, <u>**or**</u> **3** = 1 + 2, **6** = 2 + 4, **6** = 2 + 4

Note that the center cross may be rotated or inverted. Our only restriction is the **Σ** values can not be altered. Whichever way you filled the origin, diagonals summing to **σ**, determines the rest of the cross. The rest of the square is independent of how you chose to fill the cross.

This method has very few restrictions. It can be even more general than this if one wishes.

Figure 3

θ=4

1	3	1	3	1	3	1	3	1	4	4	2	4	2	4	2	4	2	45
2	4	2	4	2	4	2	4	2	3	3	1	3	1	3	1	3	1	45
1	3	1	3	1	3	1	3	1	4	4	2	4	2	4	2	4	2	45
2	4	2	4	2	4	2	4	2	3	3	1	3	1	3	1	3	1	45
1	3	1	3	1	3	1	3	1	4	4	2	4	2	4	2	4	2	45
2	4	2	4	2	4	2	4	2	3	3	1	3	1	3	1	3	1	45
1	3	1	3	1	3	1	3	**3**	**2**	4	2	4	2	4	2	4	2	45
2	4	2	4	2	4	2	4	**4**	**1**	3	1	3	1	3	1	3	1	45
3	4	3	4	3	4	**2**	**4**	**1**	**2**	**2**	**4**	2	1	2	1	2	1	45
2	1	2	1	2	1	**3**	**1**	**3**	**4**	**3**	**1**	3	4	3	4	3	4	45
4	2	4	2	4	2	4	2	**1**	**4**	1	3	1	3	1	3	1	3	45
3	1	3	1	3	1	3	1	**3**	**2**	2	4	2	4	2	4	2	4	45
4	2	4	2	4	2	4	2	4	1	1	3	1	3	1	3	1	3	45
3	1	3	1	3	1	3	1	3	2	2	4	2	4	2	4	2	4	45
4	2	4	2	4	2	4	2	4	1	1	3	1	3	1	3	1	3	45
3	1	3	1	3	1	3	1	3	2	2	4	2	4	2	4	2	4	45
4	2	4	2	4	2	4	2	4	1	1	3	1	3	1	3	1	3	45
3	1	3	1	3	1	3	1	3	2	2	4	2	4	2	4	2	4	45
45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	

The figure above shows an example of how the pattern would be applied to the 2x2 squares making the magic square. Note the solution is not unique.

Figure 4

1	3	1	3	1	3	1	3	1	4	4	2	4	2	4	2	4	2	45
2	4	2	4	2	4	2	4	2	3	3	1	3	1	3	1	3	1	45
1	3	1	3	1	3	1	3	1	4	4	2	4	2	4	2	4	2	45
2	4	2	4	2	4	2	4	2	3	3	1	3	1	3	1	3	1	45
1	3	1	3	1	3	1	3	1	4	4	2	4	2	4	2	4	2	45
2	4	2	4	2	4	2	4	2	3	3	1	3	1	3	1	3	1	45
1	3	1	3	1	3	1	3	**3**	**2**	4	2	4	2	4	2	4	2	45
2	4	2	4	2	4	2	4	**4**	**1**	3	1	3	1	3	1	3	1	45
3	4	3	4	3	4	**2**	**4**	**1**	**2**	**2**	**4**	2	1	2	1	2	1	45
2	1	2	1	2	1	**3**	**1**	**3**	**4**	**3**	**1**	3	4	3	4	3	4	45
4	2	4	2	4	2	4	2	**1**	**4**	1	3	1	3	1	3	1	3	45
3	1	3	1	3	1	3	1	**3**	**2**	2	4	2	4	2	4	2	4	45
4	2	4	2	4	2	4	2	4	1	1	3	1	3	1	3	1	3	45
3	1	3	1	3	1	3	1	3	2	2	4	2	4	2	4	2	4	45
4	2	4	2	4	2	4	2	4	1	1	3	1	3	1	3	1	3	45
3	1	3	1	3	1	3	1	3	2	2	4	2	4	2	4	2	4	45
4	2	4	2	4	2	4	2	4	1	1	3	1	3	1	3	1	3	45
3	1	3	1	3	1	3	1	3	2	2	4	2	4	2	4	2	4	45
45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	

This figure shows the pattern as it would be treated as an odd magic square.

Figure 5

185	187	229	231	273	275	317	319	1	4	48	46	92	90	136	134	180	178	2925
186	188	230	232	274	276	318	320	2	3	47	45	91	89	135	133	179	177	2925
225	227	269	271	313	315	33	35	41	44	88	86	132	130	176	174	184	182	2925
226	228	270	272	314	316	34	36	42	43	87	85	131	129	175	173	183	181	2925
265	267	309	311	29	31	37	39	81	84	128	126	172	170	216	214	224	222	2925
266	268	310	312	30	32	38	40	82	83	127	125	171	169	215	213	223	221	2925
305	307	25	27	69	71	77	79	**123**	**122**	168	166	212	210	220	218	264	262	2925
306	308	26	28	70	72	78	80	**124**	**121**	167	165	211	209	219	217	263	261	2925
23	24	67	68	75	76	**118**	**120**	**161**	**162**	**206**	**208**	250	249	258	257	302	301	2925
22	21	66	65	74	73	**119**	**117**	**163**	**164**	**207**	**205**	251	252	259	260	303	304	2925
64	62	108	106	116	114	160	158	**201**	**204**	245	247	253	255	297	299	17	19	2925
63	61	107	105	115	113	159	157	**203**	**202**	246	248	254	256	298	300	18	20	2925
104	102	112	110	156	154	200	198	244	241	285	287	293	295	13	15	57	59	2925
103	101	111	109	155	153	199	197	243	242	286	288	294	296	14	16	58	60	2925
144	142	152	150	196	194	240	238	284	281	289	291	9	11	53	55	97	99	2925
143	141	151	149	195	193	239	237	283	282	290	292	10	12	54	56	98	100	2925
148	146	192	190	236	234	280	278	324	321	5	7	49	51	93	95	137	139	2925
147	145	191	189	235	233	279	277	323	322	6	8	50	52	94	96	138	140	2925
2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	

This figure shows the actual values to be used in the magic square based on previous figure.

Figure 6

373	377	461	465	549	553	637	641	5	11	99	95	187	183	275	271	363	359	5904
375	379	463	467	551	555	639	643	7	9	97	93	185	181	273	269	361	357	5904
453	457	541	545	629	633	69	73	85	91	179	175	267	263	355	351	371	367	5904
455	459	543	547	631	635	71	75	87	89	177	173	265	261	353	349	369	365	5904
533	537	621	625	61	65	77	81	165	171	259	255	347	343	435	431	451	447	5904
535	539	623	627	63	67	79	83	167	169	257	253	345	341	433	429	449	445	5904
613	617	53	57	141	145	157	161	**249**	**247**	339	335	427	423	443	439	531	527	5904
615	619	55	59	143	147	159	163	**251**	**245**	337	333	425	421	441	437	529	525	5904
49	51	137	139	153	155	**239**	**243**	**325**	**327**	**415**	**419**	503	501	519	517	607	605	5904
47	45	135	133	151	149	**241**	**237**	**329**	**331**	**417**	**413**	505	507	521	523	609	611	5904
131	127	219	215	235	231	323	319	**405**	**411**	493	497	509	513	597	601	37	41	5904
129	125	217	213	233	229	321	317	**409**	**407**	495	499	511	515	599	603	39	43	5904
211	207	227	223	315	311	403	399	491	485	573	577	589	593	29	33	117	121	5904
209	205	225	221	313	309	401	397	489	487	575	579	591	595	31	35	119	123	5904
291	287	307	303	395	391	483	479	571	565	581	585	21	25	109	113	197	201	5904
289	285	305	301	393	389	481	477	569	567	583	587	23	27	111	115	199	203	5904
299	295	387	383	475	471	563	559	651	645	13	17	101	105	189	193	277	281	5904
297	293	385	381	473	469	561	557	649	647	15	19	103	107	191	195	279	283	5904
5904	5904	5904	5904	5904	5904	5904	5904	5904	5904	5904	5904	5904	5904	5904	5904	5904	5904	

Just a reminder that for any magic square,

$x \to \alpha x + \beta$ will still give us a magic square. In this case, $\alpha = 2, \beta = 3$.
$x \to 2x + 3$

$\Sigma_{new} = \alpha\Sigma_{original} + \beta n$ $\quad n$ = size of magic square.

$5904 = 2(2925) + 3(18)$